Using coherencies to examine network evolution and co-evolution

George A. Barnett [1,*,2]

Email gbarnett@ucdavis.edu

Ke Jiang [1]

Jesse R. Hammond [2]

[1] Department of Communication, University of California, Davis, CA, 95616 USA

[2] Department of Political Science, University of California, Davis, CA, 95616 USA

Abstract

This paper explores the use of spectral or frequency domain analysis to examine network evolution and co-evolution. Given a sufficient number of measurements of a valued network over-time, a matrix (**K**) may be created, where κ_{ij} is the squared coherency between nodes *i* and *j*. Since **K** is a matrix of relations, it may be analyzed like any other network to determine which nodes co-evolve, and which change independently. To illustrate this approach, cross-spectral analyses were conducted among 250 pairs of nation-states based on terrorism news coverage (2000–2012) to create matrix **K**. Then, a network analysis was conducted. The results indicate that the United States was the most central country, and that the nations co-evolved in two large groups (the Middle East and East Asia), and numerous smaller ones. The proposed method is extended to include exogenous variables (terrorist events and oil prices) and to two or more networks to examine network co-evolution. The method's application to network co-evolution is demonstrated with additional data on networks of co-association in terrorism news coverage (2011–2013) and a comparable data set on the coverage of the Arab Spring (2011–2013).

Keywords

Network evolution and co-evolution
Spectral or frequency domain analysis
Coherence

1. Introduction

The study of how social networks evolve and how two or more networks co-evolve time has become of increasing interest to network scholars (Brandes and Corman 2003; Carley 1999; Monge and Contractor 2003; Monge et al. 2008; Weber and Monge 2011). Time series analysis may be used (Barnett et al. 1991; Barnett et al. 1993; Gong et al. 2011) to examine the evolution of the networks and spectral or frequency domain analysis may be used to investigate the co-evolution of networks (McCulloh 2009; McCulloh et al. 2012). Time series analysis and spectral analysis allow for the precise description of how the relationships within networks vary with respect to time. Typically, Fourier spectral analysis has been used to identify cycles in longitudinal data (McCulloh et al. 2012). For example, Elbert and Barnett (2006) used spectral (frequency domain) analysis to confirm a seasonal cycle in Canada's inter-provincial migration network.

Typically, network evolution, how networks change with respect to time has been examined by looking at discrete changes in the state of binary graphs or adjacency matrices (Toivonen et al. 2009; Snijders et al. 2010). However, there is no reason that the measurement of the relationship among a pair of nodes must be limited to its presence or absence (Butts, 2009). For example, it might be measured as the number of interactions between i and j, or some other value of the attribute of a relationship between nodes (Harary 1969).

AQ1

This paper proposes a method that examines changes in the relations among a set of individual nodes (edges), network evolution, with valued data measured continuously with respect to time. These procedures are then generalized to two or more networks with equivalent nodes for the examination of network co-evolution. The paper is divided into six sections. Following the introduction (Sect. 1), it describes a method to examine network evolution in Sect. 2. An illustrative example is provided, which includes how the data were mined from the LexisNexis news archive and results of analysis. The method is extended to include exogenous variables in Sect. 3. This is followed by a discussion of network co-evolution (Sect. 4). The limitations of the proposed method and future research are next discussed in Sect. 5. The paper ends with a summary and conclusions (Sect. 6).

AQ2

AQ3

2. Network evolution

Given a sufficient number of measurements of a valued network at different points in time, the edges' changing relationships can be represented through spectral or frequency domain analysis, leading to this research's methodological contribution. That is, the use of Fourier spectral analysis to study network evolution and co-evolution. By network evolution, we simply mean how the overall network and its various nodes change with respect to time. Network co-evolution refers to two or more networks changing synchronously or asynchronously over an interval of time.

Spectral analysis or frequency domain analysis or spectral density estimation is the procedures that allow the decomposition of a complex over-time process into simpler parts. Many physical and social processes may be described by the sum of individual frequency components (amplitude, powers, intensities, or phases). Any process, including network evolution, which quantifies the various amounts, versus frequency may be called spectral analysis. Once, the non-stationarity in two or more processes is removed, the changes in the processes may be correlated to quantify the ability to predict the second process knowing the first.

Past research on network evolution has typically examined how group or community structure and membership change (Backstrom et al. 2006). For example, Gliwa et al. (2013) predict community evolution (growing shrinking, splitting, merging and dissolving) from network properties (centralization, density, size and cohesion). Their focus is on community or group evolution rather than the evolutionary changes in the entire network and the relations among the changes of individual nodes. Similarly, Singhal et al. (2014) examined the evolution and co-evolution of communities in a multi-relational (trade and trust) network based on their connectivity.

Toivonen et al. (2009) compared two different models of network evolution, NEMs, in which the addition of new links is dependent on the (typically local) network structure and NAMs whose links are generated based only on nodal attributes. In both cases, they focused on network growth or shrinkage until the network stabilized with non-directional binary data. The current paper differs from the Toivonen et al. research in that it focuses on networks of a given size with valued link data.

Gong et al. (2011) used time series analysis to discover that network structure can reveal the diversity and novelty of the information being communicated in a network. Networks with a higher conductance (the potential for directed information flow between nodes) in link structure exhibit higher information entropy, while unexpected network configurations are indicative of higher information novelty.

AQ4

Skillicorn et al. (2014) address the issue of network dynamics, especially as they apply to valued graphs, through the use of spectral techniques that embed a graph in a geometric space. Similar approaches have been taken by Barnett and Rice (1985) and Seary and Richards (2003). Their approach tracks the similarity of nodes over-time to determine the global trajectory of individual nodes across time. It differs from the approach advocated here because our goal is to determine which nodes change (co-evolve) in a similar way, that is, have similar patterns of change, and which change independently from the other nodes. This is accomplished by removing the overall time trend, the non-stationary from the data, and then determining the cross-covariance function between the series.

2.1. Data and methodology

A matrix (**K**) may be created, where κ_{ij} is the squared coherency (κ_{ij}^2) among all pairs of nodes i and j. Coherence squared (κ^2) can be defined as analogous to the squared correlation coefficient (Gottman, 1979). Coherence conveys how well correlated two processes are as quantified by the cross-correlation function. It is a measure of association between two time series after removing the overtime or non-stationary in the data. This is typically accomplished after decomposing the physical or social process using spectral analysis or frequency domain analysis on the measures of each process. The time trend is removed from the data because the focus is on the degree to which the individual nodes' changes are related, how they co-evolve, rather than the general longitudinal trend in the data. Matrix **K** is symmetric ($\kappa_{\mathrm{ij}}^2 = \kappa_{\mathrm{ij}}^2$) and its diagonal, $\kappa_{\mathrm{ij}}^2 = 1$. Since **K** is a matrix of relations, a network analysis may be conducted to determine which set of nodes co-evolves and which changes more or less independently.

To illustrate this approach to examine network evolution, data were gathered using the LexisNexis (http://www.lexisnexis.com) news archive to construct a network based on the co-occurrences of country names in news articles. Segev et al (2013) used a similar approach to examine structural trends in international news.

To create a data set representative of the global news network, rather than American or Western European media, a set of news sources was drawn from throughout the world. The sources included two newswire services (Agence France-Presse (France), Xinhua News Service (China)), and seven newspapers (The Guardian (United Kingdom), The New York Times (United States), The Straits Times (Singapore), The Daily Yomiuri (Japan), The Hindustan Times (India), The China Daily Service (China), and The Moscow Daily (Russia)). Coder restrictions limited the sources to English-only news services. Likewise, data availability issues restricted the use of African and Latin American news sources. However, the selection of sources represents an effort to address potential issues of geographic, political, or issue-centric bias that might arise with a focus on only one or two news sources.

To maximize the sample size, an extremely general search string was used, retrieving any news article from the list of sources that included some permutation of the root word terror, such as terrorist, terrorize, terrorism. While this does increase the expected rate of 'false positives', entries in which the keyword appears but do not actually deal with 'terrorism', it is not expected to produce any systematic bias in the results. The end result may be somewhat noisier, but still a fundamentally accurate depiction of news media coverage and discussion of terrorism. Data were collected from January 2, 2000 through December 29, 2012. The final data set included 529,503 unique articles from nine different international news sources.

The network of terrorism news coverage was examined as it pertains to the international state system (Barnett et al. 2013). Although international terrorism is by definition committed by non-state actors, states are commonly associated with terrorist groups. Terrorist acts themselves are generally described as being committed against states, as their targets or rhetoric often reflects animosity towards the target state. Terrorist acts themselves are also commonly associated with states that provide (or are accused of providing) safe haven for violent groups or shelter from international reprisals.

To capture the international terrorist network, the database of news articles was mined to create a symmetric matrix (**X**) of state co-occurrences, which recorded the number of times two states are listed in the same article. For example, an article mentioning both Russia and Azerbaijan would result in a "1" in the matrix entry x_{ij} for those two states. It should be noted that this is a valued network, so any single article would increase the value of the matrix entry x_{ij} by one. State nodes are connected if they co-occur at least once in an article, and the strength of their connection is equal to the number of co-occurrences throughout the whole database. This is a fairly crude metric, but it has the potential to uncover patterns in the international terrorism news network. Finally, the daily entries were aggregated to the weekly level to reflect the typical news cycle and to avoid null entries due to sparse data since some days contained few articles about terrorism.

This resulted in 624 measures of the terrorism news network (52 weeks times 12 years). Barnett et al. (2013) examined the same database. However, in that case the data were aggregated to the annual level (2001–2010).

Initial analysis within the international terrorism news network found that the overall network density, average link strength and the number of triads increased significantly after the terrorist attacks of September 11, 2001 (Barnett and Hammond 2013). Afterward, density and mean link strength steadily declined, but with a spike in 2005, likely related to the Iraqi insurgency attacks on multinational troops and other deviations from a monotonic decline due to the coverage of specific terrorist events [Madrid train bombing (March 2005), London tube bombing (July 2005), Mumbai terrorist attacks (September 2008)]. Changes in the network position of individual countries reflected activities affecting international ties to those nations. For example, Iraq became more central during the United States intervention and less central as US troops withdrew; Afghanistan's centrality increased during the military surge; Syria remained peripheral until the onset of its civil war during which it became more central; and Nigeria was peripheral except during the coverage of the underwear bomber incident in late 2009.

Cross-spectral analysis revealed that network density changed as a function of the number of terrorist events, and that change in an individual country's network position was related to the number of terrorist events that took place within the country. Further, while changes in one country's position in the network were strongly related to changes in other countries, others were weakly related. For example, while the squared coherency (κ^2) for India and Pakistan was 0.859, two countries strongly associated with one another about terrorism, it was only 0.220 for Denmark and Afghanistan, two nations who have not been connected regarding terrorism. These findings suggest that the proposed methods are on the surface valid. As a result, it was decided to pursue the proposed model.

To determine the co-evolution of the nodes in the network, cross-spectral analyses were conducted among all possible pairs of the 250 states (31,125 pairs) in the terrorism news network to create matrix **K** (Gottman 1979; Granger and Hatanaka

1964; Jenkins and Watt 1968). Two time series (x_{ijt} and x_{ikt}) may be correlated with one another at various time periods (in this case weekly), and similar to regression analysis a coefficient, coherence squared (κ^2) can be defined that is analogous to the squared correlation coefficient (Gottman 1979). The spectrum of a series (x_{ijt} for t_1–t_{624}) is the Fourier transform (for periodic data) of the autocovariance function of the series, and the cross-spectrum is the Fourier transform of the cross-covariance function between the series (x_{ijt} and x_{ikt}) (Bloomfield, 1976) . The strength of association between the series may be determined by examining the coherence between spectra at various length periods after removing the overtime trend or non-stationary in the data.

The slope of the phase spectrum may be examined to determine the time lag between the time series, helping to determine the direction of causality. The lag is equal to the slope of the phase spectrum. A positive or negative slope indicates how much the changes in one series precede (or follow) another.

Matrix **K** was created using the R Packages ‘foreign’ (R Core Team 2014), ‘tm’ was employed for text mining (Feinerer and Hornik 2014; Meyer et al. 2008), ‘Snowball’ was employed for text mining (Hornik 2014). ‘igraph’ was used to perform network analysis (Csardi and Nepusz 2006), time series analysis (spectral and coherency analysis) was performed with ‘astsa’ (Stoffer 2014) and ‘forecast’ (Hyndman et al. 2014), ‘tnet’ was employed for the analysis of weighted, two-mode, and longitudinal networks (Opsahl 2009), and ‘PCIT’ (Watson-Haigh et al. 2010) to identify meaningful correlations to define edges in the network of coherencies. Then, a network analysis was conducted with matrix K using UCINET-6 (Borgatti et al. 2002). The network diagrams were drawn with Gephi (Bastian et al. 2009).

AQ5

2.2. Results

The most central country in terms of degree (the sum of the valued links) in the evolution network was the United States, followed by Germany, Dominican Republic, France, Japan, Russia, Belgium, and the United Kingdom (Freeman

1979). All of these countries with the exception of the Dominican Republic play a prominent role in international relations. It should be noted that these results describe the similarity in the degree of change in the news network and not their centrality in international relations. Because these countries were typically mentioned in the same stories they were similar in their patterns of change. The Dominican Republic while mentioned less frequently changed in the network in the same manner as the other more significant nations. They were all about twice as central as the mean centrality (51.34, s.d. = 32.90). Those countries with an average squared coherence of 0.3 or greater are presented in Table 1. When using squared coherence as the measure of link strength, centrality is equal to the sum of the squared coherences for each node. Thus, the overall network changed in a pattern that resembled the changes for the United States, Western Europe, Japan and Russia.

Table 1

Centrality in the evolution network

United States	104.253 Could you right justify the numbers in this table?	Hong Kong	92.957
Germany	104.128	Australia	92.189
Dominican Republic	103.188	Czech Republic	92.062
France	102.871	Turkmenistan	92.038
Japan	102.240	Iran	91.894
Russia	101.044	Mexico	91.323
Belgium	100.904	Norway	90.913
United Kingdom	100.766	South Africa	90.881
Tanzania	100.112	Thailand	90.838
Ireland	100.005	Indonesia	90.445
El Salvador	99.818	Ghana	90.126
Saudi Arabia	98.943	Bangladesh	88.777

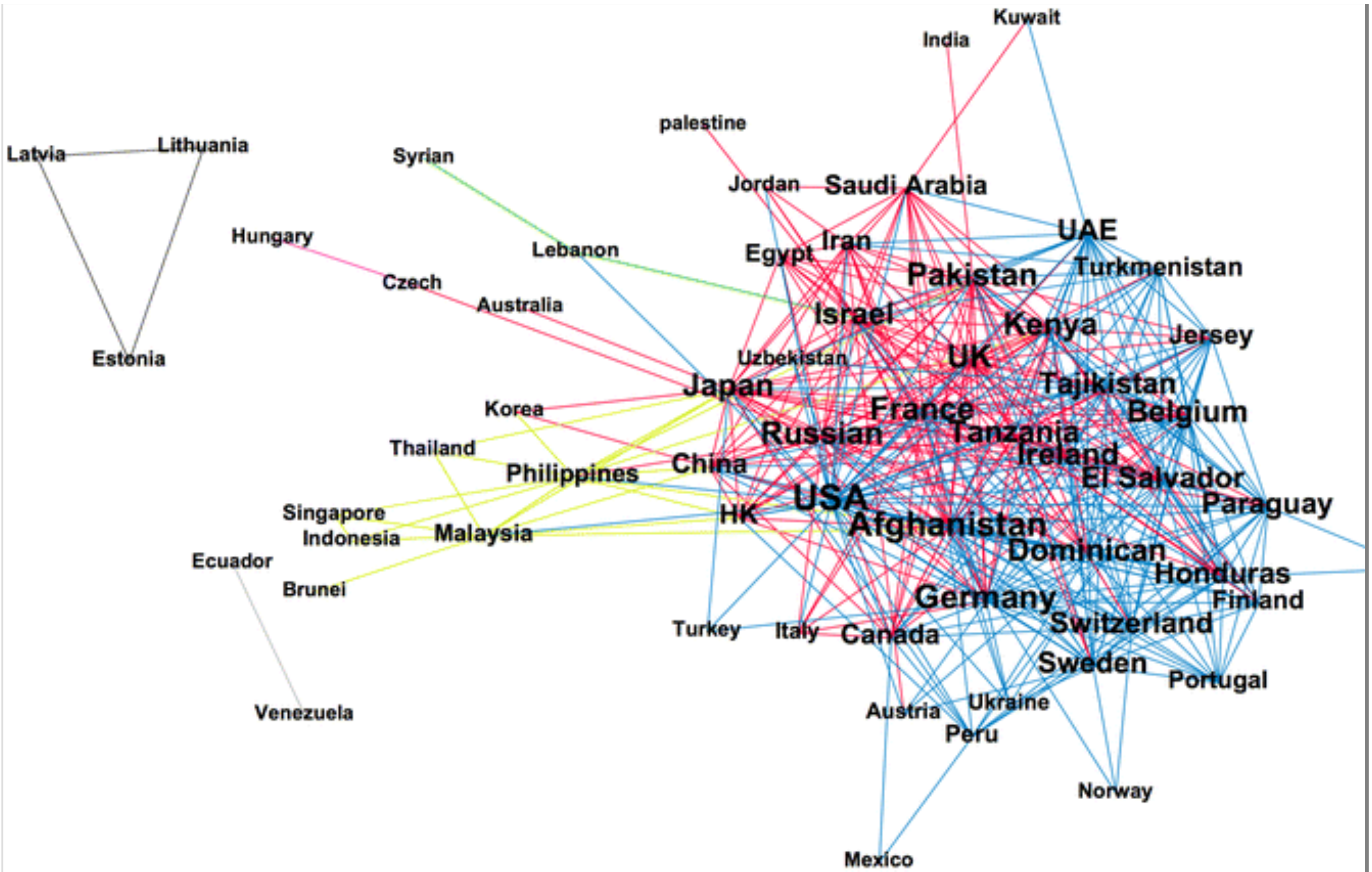
Netherlands	98.475	Spain	88.335
Afghanistan	98.239	Argentina	87.098
Canada	97.904	Nauru	86.764
Honduras	97.223	India	86.714
Philippines	97.100	Palestine	86.405
China	96.823	Albania	86.396
Malaysia	96.486	Panama	85.945
Italy	96.162	Lebanon	85.773
Kenya	95.798	Singapore	85.463
Turkey	95.658	Croatia	85.095
United Arab Emirates	95.582	Zimbabwe	84.365
Jordan	95.317	Sudan	83.454
Austria	95.300	Gambia	82.526
Paraguay	95.240	Ecuador	81.977
Switzerland	95.232	Venezuela	81.335
Tajikistan	95.202	Viet Nam	80.768
Pakistan	94.677	Poland	79.994
Hungary	94.359	Nigeria	79.616
Sweden	94.156	Syria	79.570
Egypt	94.135	Iraq	78.610
Uzbekistan	93.883	Morocco	77.299
Israel	93.864	Liechtenstein	76.493
Peru	93.752	Yemen	75.924
Portugal	93.592	Cyprus	75.652
Finland	93.484	Mean (Overall)	51.340

Kuwait	93.351	Standard deviation	32.900
Ukraine	93.331		
South Korea	93.137		
Taiwan	92.989		

To determine the co-evolution of individual states in the network, a hierarchical cluster analysis was conducted to see which countries formed groups based on the strong relationships among their patterns of change (Aldenderfer and Blashfield 1984). This resulted in two large groups and numerous smaller ones. The larger were from the Middle East and included Syria, Lebanon, Palestine, U.A.E., Israel, Jordan, Saudi Arabia, Egypt, and Kuwait. It was centered about Israel and Saudi Arabia. Also, there was an East Asian group composed of Singapore, Indonesia, Thailand, Korea, Australia, Philippines, Malaysia, Hong Kong, China, and Japan, centered about Malaysia, Philippines and Japan. As indicated in Table 1, the members of both of these groups are central in the overall change network. The smaller groups of states that co-evolved in the network were the Baltic States (Slovakia, Estonia, Latvia and Lithuania), the Nordic countries (Sweden, Finland and Norway) plus Portugal, Latin America (Mexico, Peru, Venezuela, Columbia and Ecuador) and a dyad (Czech Republic and Hungary). A graphic representation of the network resulting from the squared coherencies is presented in Fig. 1. It was produced using Gephi (Bastian et al. 2009). The minimum link strength (coherency squared) required for a line was 0.886, which was the mean (κ^2 = 0.206) plus 2.75 standard deviations (0.235), a rather strong relation.

Fig. 1

International terrorism news network. The ties are based on the squared coherency among the nodes. κ_{ij}^2 > 0.866 (mean + 2.75 s.d.) required for a link. The *different clusters* are represented by *different colors*



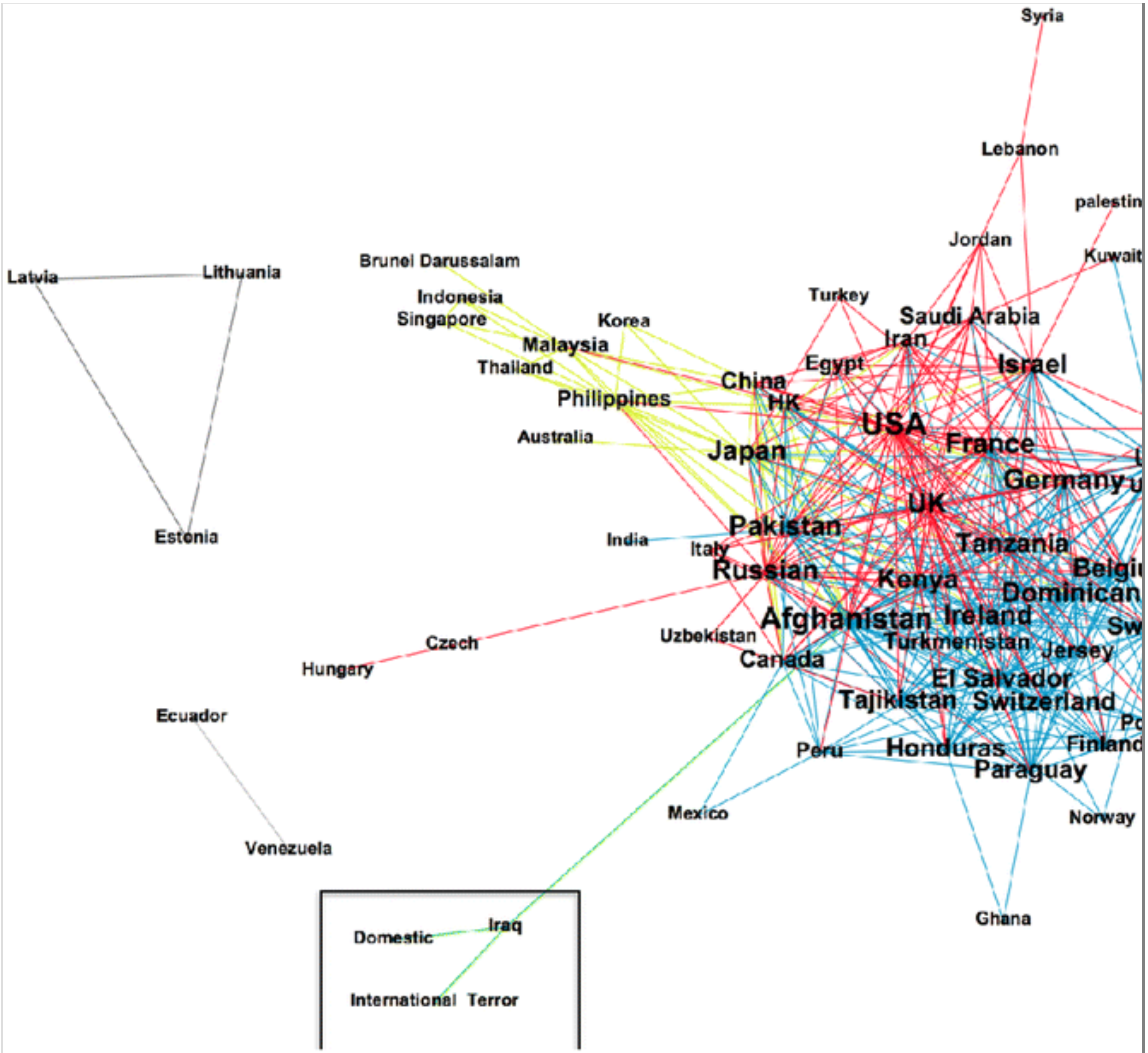
3. Extension to exogenous variables

Because the relations among the nodes were established through Fourier spectral analysis, patterns of co-occurrence over-time, exogenous variables may be added to the analysis. For example in this case, the relationship (κ^2_{ix}) between the number of terrorist events and the coverage of each country may be included in a matrix **K***, where **x** is a vector of squared coherences. Thus, **K* = K + x.** This allows for the determination of which states' positions in the international network covary most closely corresponds with actual terrorist events. Data on terrorist events were gathered from the National Consortium for the Study of Terrorism and Response to Terrorism's (START) Global Terrorism Database (www.start.umd.edu/gtd). For these data, the mean coherency between terrorist events and changes in a country's degree centrality was 0.159 and the standard deviation 0.132. The coherency between the change in overall number of terrorist events and the overall change in network density was $\kappa^2 = 0.712$. Iraq had the highest correspondence with terrorist events. Other states whose network position and terrorist events co-varied greater than two standard deviations plus the mean were Brunei, Bulgaria, Fiji, Germany, Jordan, Kuwait, Laos, Malaysia, Morocco, Mozambique, and Timor.

Figure 2 shows the coherency-squared network **K*** with the addition of two exogenous variables, the number of domestic and the number of international terrorist events. To facilitate interpretation κ_{ij}^{2} was set at 0.866 (for a link. Note the link between Iraq and the exogenous variables at the bottom center of the figure. This indicates that Iraq's changing position in the network is most closely associated with terrorist events. Most central in **K*** is the United States, followed by Greece, Afghanistan, Russia, Japan, Canada, Georgia, Israel, the United Kingdom, and the U.A.E. Although there is some similarity between the networks produced by **K** and K*, there are also differences in structure due to the addition of the exogenous variables. For example, only eight of the ten most central countries in **K** are also among the most central in **K***. By taking the events into account, Greece and Georgia become relatively more central.

Fig. 2

The International terrorism news network with exogenous variables included. The ties are based on the squared coherency among the nodes. A squared coherency (κ_{ij}^{2}) > 0.866 is required for a link to be displayed. The exogenous variables, domestic and international terrorism are in a *box*



4. Network co-evolution

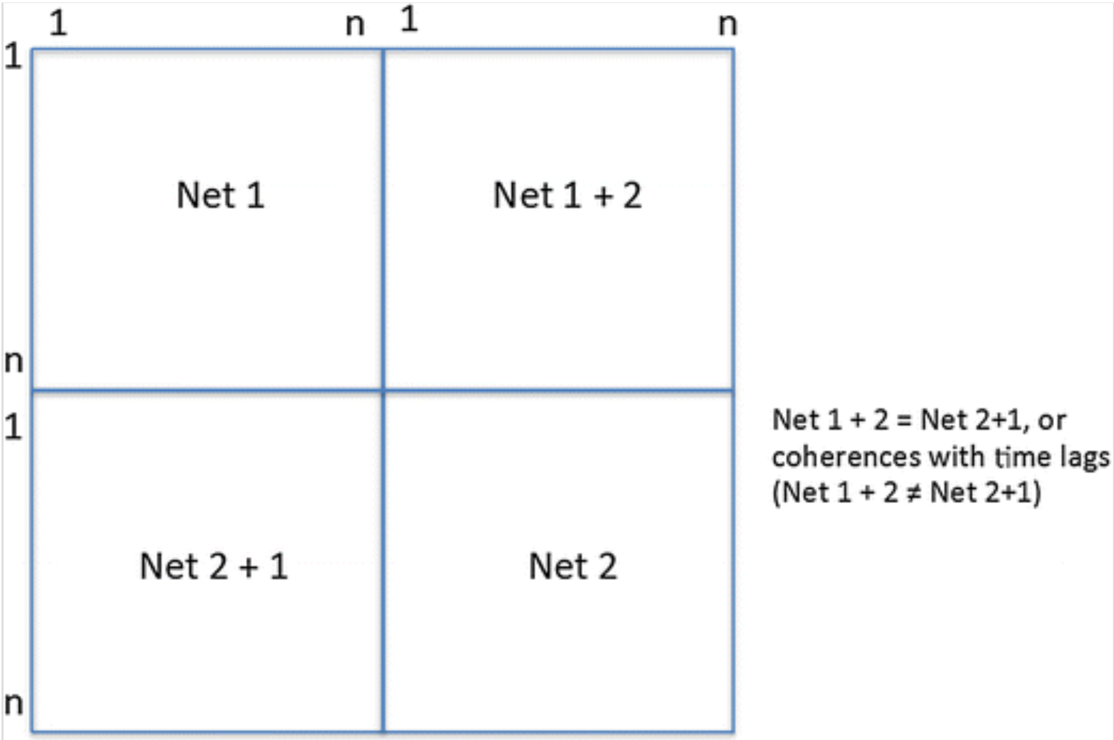
4.1. Theory and method

The co-evolution of two (or more) networks with the same nodes and equivalent time frames may also be examined through the use of cross-spectral analysis. To study co-evolution at the network level, a time series of network measurements may be correlated, such that the corresponding cells of sociomatrix at time t are correlated with the same network at $t + 1$ for the entire series, creating a n − 1 vector of correlations, **r**. This is repeated for each network. These two (or more) vectors (time series ($\mathbf{r}_i$ and $\mathbf{r}_j$) may be Fourier spectral analyzed producing a squared coherence, κ^2, which describes the extent of co-evolution for this pair of networks. The slope of the phase spectrum may be examined to ascertain the time lag between the time series to determine the direction of causality.

The co-evolution of two (or more) networks may also be examined at the nodal level. Consider a supra-matrix **K**** composed of two minor matrices **K1** and **K2**, composed of the squared coherencies for each individual network, and two additional minor matrices **K12** and **K21,** composed of the squared coherencies between the corresponding cells of **K1** and **K2**. This is illustrated in Fig. 3. Where the two networks evolve simultaneously without the changes in one network causing changes in the second, there is no phase shift, **K12** = **K21.** Where a time lag exists between the networks, and there exists the potential for spillover with the changes in one network causing changes in the other, and **K12** ≠ **K21.** Since **K**** is also a matrix of relations, a network analysis may be conducted to determine how the nodes from **K1** co-evolve with those nodes from **K2** and which change over-time more or less independently. The squared coherencies of various time lags between **K1** and **K2** may be inserted in **K**** to ascertain causality and the over-time relations among specific nodes from the different networks.

Fig. 3

Data structure for the extension of the model to network co-evolution



AQ6

4.2. Data

To illustrate this approach to examine network co-evolution, data were gathered using the LexisNexis (http://www.lexisnexis.com) news archive to construct a network based on the co-occurrences of countries in news articles for the Arab Spring, using the phrases "Arab Spring," "Arab Revolution," "Arab Uprising," and "Arab Awakening" as key phrases. Data were gathered from January 2011 to December 2013. The same nine media sources were searched as for the terrorism news network and the terrorism data set was extended through 2013. The entries were aggregated to the weekly level. Thus, there were 156 time points for both data sets (52 weeks times 3 years). 7182 stories were mined to construct the Arab Spring network, and 96,854 stories for the terrorism network.

4.3. Results

The squared coherence (κ^2) across the two networks was 0.692 ($p < 0.000$), with the coverage of the Arab Spring lagging the coverage of terrorism by one week. While the two networks co-evolved, they differed in structure. The Arab Spring network had a core-periphery structure with 17 countries at the core and a Gini-coefficient of 0.847 (Borgatti and Everett 1999). The terrorism network was composed of a single cluster centered about the United States and Afghanistan that included many more countries. Its Gini-coefficient was 0.824, indicating somewhat less centralization than the Arab Spring network. Table 2 shows the most central countries in the two networks.

Table 2

Degree Centralities for Arab Spring and Terrorism Networks

	Arab Spring			Terrorism			
	Rank	**Degree**	**Share**	**Rank**	**Degree**	**Share**	**Coherence**
United States	1	20573 Could you right justify the numbers in this table and insert commas if possbile?	0.079	1	153711	0.107	0.108
Syria	2	14255	0.055	3	54538	0.038	0.124
Egypt	3	13989	0.054	13	28514	0.020	0.173
Libya	4	13569	0.052	12	32786	0.023	0.342
Tunisia	5	10730	0.041	26	13944	0.010	0.167
Iran	6	8458	0.033	8	41663	0.029	0.235
Israel	7	7929	0.031	10	36370	0.025	0.194
China	8	7376	0.028	9	38253	0.027	0.048
Iraq	9	7358	0.028	7	42039	0.029	0.301
United Kingdom	10	7241	0.028	5	44614	0.031	0.137
France	11	7202	0.028	6	43624	0.030	0.120
Russia	12	6488	0.025	11	35309	0.025	0.067
Saudi Arabia	13	6196	0.024	18	21150	0.015	0.205

Yemen	14	5978	0.023	17	22282	0.016	0.276
Afghanistan	15	5575	0.021	2	56698	0.040	0.500
Turkey	16	5141	0.020	15	26219	0.018	0.124
Palestine	17	4865	0.019	19	20277	0.014	0.441
Bahrain	18	4205	0.016	<40	6590	0.005	0.174
Pakistan	19	4197	0.016	4	51891	0.036	0.517
Lebanon	20	4192	0.016	23	18856	0.013	0.159
Jordan	21	4090	0.016	<40	13585	0.009	0.127
Qatar	22	3570	0.014	<40	11563	0.008	0.194
Germany	23	3556	0.014	16	22385	0.016	0.074
Japan	24	3481	0.013	21	19060	0.013	0.259
Morocco	25	3158	0.012	<40	7543	0.005	0.207
India	26	3109	0.012	14	27029	0.019	0.065
Italy	27	2892	0.011	25	14452	0.010	0.079
Greece	28	2338	0.009	<40	6457	0.004	0.423
UAE	29	2153	0.008	<40	6210	0.004	0.144
Kuwait	30	2036	0.008	<40	6516	0.005	0.195
Algeria	31	2000	0.008	29	11563	0.008	0.054
South Africa	32	1922	0.007	36	10120	0.007	0.200
Spain	33	1915	0.007	32	11019	0.008	0.071
Sudan	34	1653	0.006	38	9556	0.007	0.469
Mean		1038.58	0.004		5740.5	0.004	0.114
S.D.		2531.67	0.010		13615.05	0.009	0.142

AQ7

The evolution of the two networks may be examined with respect to changes in exogenous variables. In this case, the relationship between the number of terrorist events and oil prices and the changes in the two news coverage networks were examined. The squared coherences among the four time series are presented in Table 3. Arab Spring coverage was only weakly related to terrorist events ($\kappa^2 = 0.067$) and oil prices ($\kappa^2 = 0.190$). Terrorism news coverage was more strongly related to terrorist events ($\kappa^2 = 0.329$) and oil prices ($\kappa^2 = 0.210$). While there was no significant lag between terror events and changes in the two networks, changes in oil prices preceded changes in both networks by five weeks.

Table 3

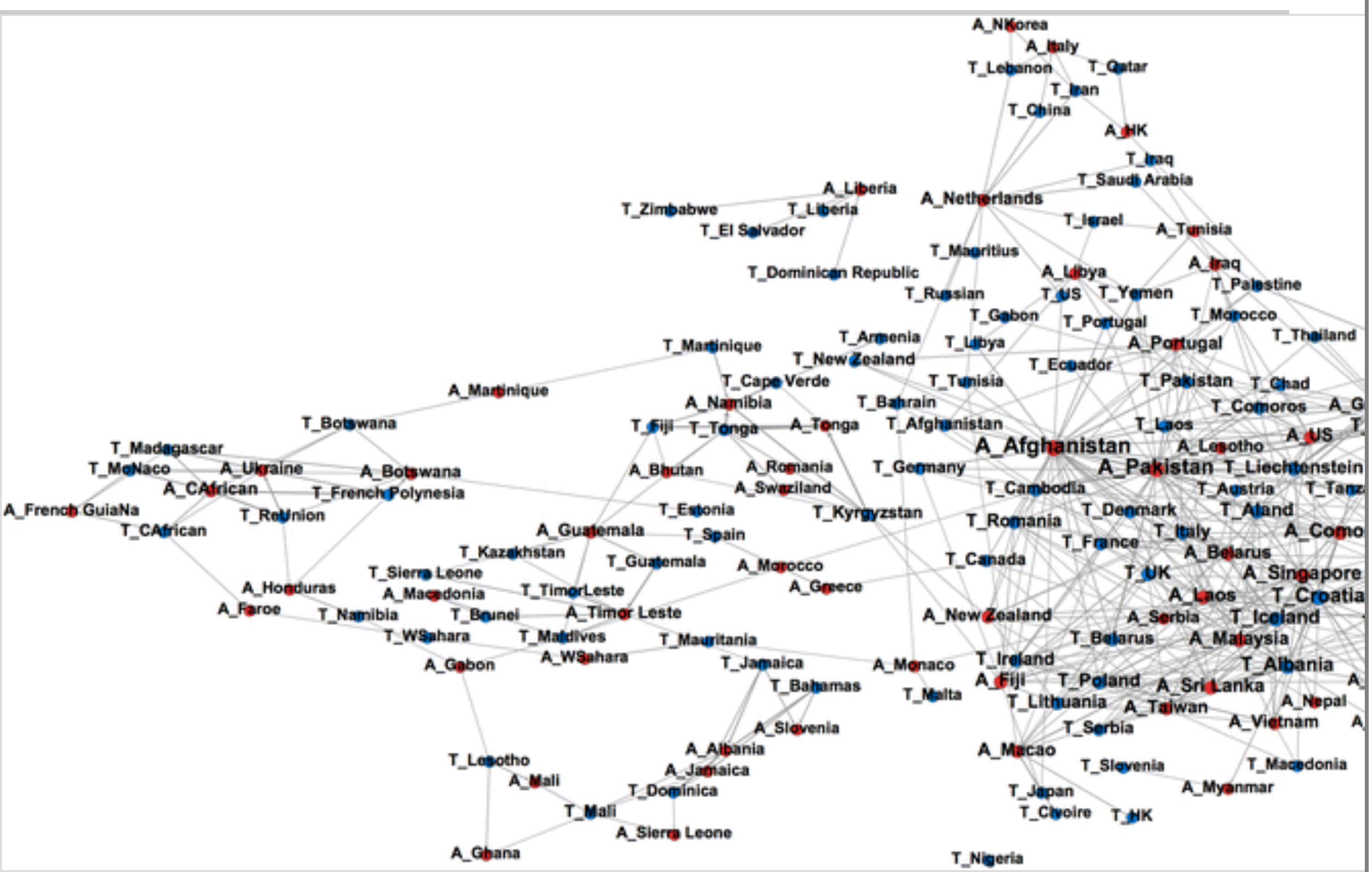
Squared Coherences among the Arab Spring and Terrorist News Networks, Oil Prices and Terrorist Events

	1	**2**	**3**	**4**
1. Arab Spring	1.000	1–2 1st	No lag	5–4 1st
2. Terrorism	0.692	1.000	No lag	5–4 1st
3. Terror Events	0.067	0.329	1.000	No lag
4. Oil Prices	0.190	0.210	0.029	1.000
Lower triangle squared coherences, *upper triangle* lag				

Figure 4 displays the central portion of the co-evolution of the two networks as a two-mode network. Only the network's most central nodes are displayed because the full network would be unintelligible due to the large number of nodes resulting from displaying both networks simultaneously (500 nodes). The blue nodes are the terrorist network and the red are the same nodes in the Arab Spring network. Lines are drawn for those relationships stronger than the mean (0.045) plus four standard deviation (0.064) units. The darker the line the stronger is the relationship. These lines show which countries in the terrorism network changed in a similar manner simultaneously with those in the Arab Spring network. Many of the strongest ties are for a country with itself (Afghanistan, Greece, Pakistan, Palestine and Sudan) in the two networks, indicating that as they shifted position in the terrorist network they also changed position in the Arab Spring network. The last column in Table 2 shows the squared coherence for each country in both networks. Note that Egypt's strongest tie is its Arab Spring position and Yemen's in the terrorist network. Thus, stories about Yemen and terrorism covary strongly with Egypt's Arab Spring news. Also, some of the strongest relationships are Afghanistan and Pakistan's Arab Spring ties to Western Europe (Italy, U.K., Germany and France) about terrorism. This suggests that there is "spillover" from the Arab Spring coverage to the news stories associated with terrorism.

Fig. 4

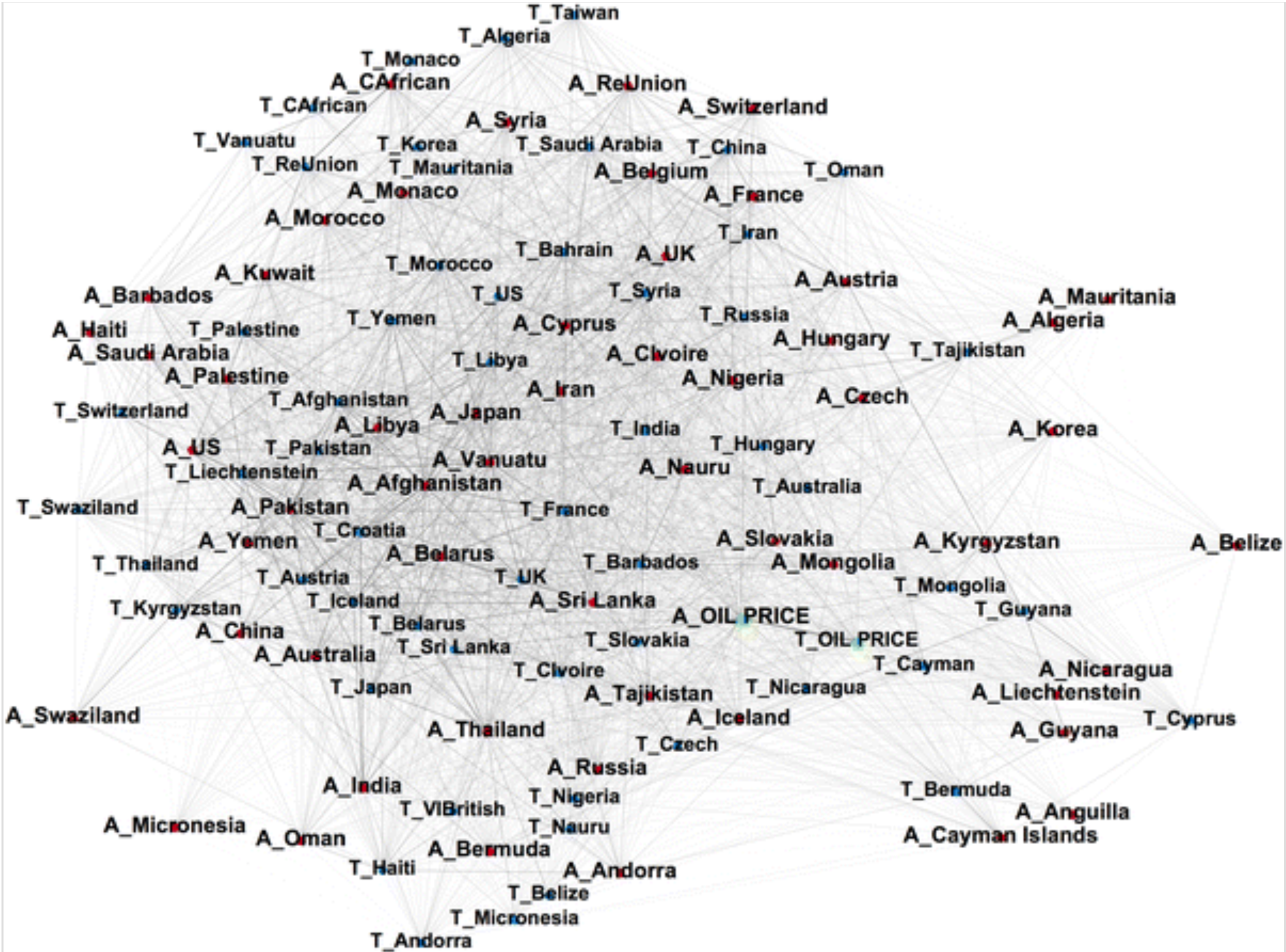
Two-mode network representing the co-evolution of the Arab Spring and terrorism news networks. The blue nodes represent the terrorist network and the *red* the *same nodes* in the Arab Spring network. A squared coherency (κ_{ij}^{2}) > 0.3 is required for a link to be displayed



As with the evolution of a single network, exogenous variables may be added to examine their impact on network co-evolution. The co-evolution of both networks as a two-mode network was examined with oil prices added as an exogenous node. Once more, the strongest ties were for the certain countries with themselves (Afghanistan, Pakistan, Palestine and Libya) in the two networks, indicating that as they changed in terrorist network, they changed in the Arab Spring network. However, the strongest relationship in this network was by definition, oil prices, which has same values for both modes. There were 27 links between oil prices and individual countries in the Arab Spring network ($\kappa^2 \geq 0.100$). Oil prices appear to have a weaker impact on the Arab Spring than the terrorist network. Also, oil prices have stronger relationships with specific nodes in the terrorist network with 39 links ($\kappa^2 \geq 0.100$), including China, Russia, Syria, Iran, Morocco, Saudi Arabia, Yemen, and Pakistan. Finally, as seen in Fig. 5, oil prices are relatively peripheral the co-evolution network, somewhat distant from the most central nations—United States, Afghanistan, Pakistan, Palestine and Libya.

Fig. 5

Two-mode network of the co-evolution of the Arab Spring and terrorism news networks with an exogenous variable (oil prices) included. The *blue nodes* represent nodes in the terrorist network and the red the same nodes in the Arab Spring network. Oil prices are represented in *green*. A squared coherency (κ_{ij}^2) > 0.100 is required for a link to be displayed



5. Limitations and future research

One problem with the proposed methods is that a large number of time-varying measurements, more than is typical for network analyses, are required for stable and robust results. Bad estimates of the coherency coefficient can result if the record length is too small due to the lack of stability in the time series (Jenkins and Watts 1968). As many as 400 points in time may be required depending on such features as the slope of the spectrum across a range of frequencies and the amplitude of the measured edge attributes. Thus, the data used to demonstrate the co-evolution of the two networks may have too few time points to say precisely how the network co-evolved.

A second limitation of this research is that the network and the model of evolution were based on non-directional data ($x_{ijt} = x_{jit}$). This is particularly a problem when considering the time lag (phase shift) within a given network and with the co-evolution of networks where changes in one network spillover to another. To address this problem, a directional asymmetric network may be generated at each point in time, and from which an asymmetric matrix **K** with ($\kappa_{\text{ij}}^2 \neq \kappa_{\text{ij}}^2$) calculated.

A third problem with this research is that it only describes the evolution and co-evolution of the networks, rather comparing the descriptions against a set of null hypotheses. Future research should model the evolution of networks against a set synthetic data where the results are known, allowing one to make a case that the approach finds the expected structure. Alternatively, we should model a known empirical case to determine the validity of the proposed procedures. Currently, data on network evolution and co-evolution are being generated using all three approaches—simulation, experimentally and mined data that will allow us to determine the viability of the proposed method.

Future research is planned to assess the viability of the proposed model. Jiang et al. (2014) used this model to examine the co-evolution of cultural symbols in semantic networks of coverage of the Arab Spring from The Associated Press and Xinhua News Agency. They found that the changes of the eigenvector centralities of the negative concepts, Crisis and Unrest, which reflect Chinese opinions toward the Arab Spring in the Xinhua News Agency were likely to lead the changes in the eigenvector centralities of the symbol Free in The Associated Press in the same direction about one month later. Additional data sets where the relations among the nodes are directional will be identified and the model examined with these networks. However, it is difficult to gather or obtain existing network data sets with sufficient points in time using fine grain measures of the strength of ties among the nodes.

6. Summary and conclusions

In summary, the major contribution of this paper is that it provides ~~and~~ an initial exploration of the use of Fourier spectral or frequency domain analysis to examine network evolution and co-evolution. Given the measurement of a valued network at a large number of points in time, a matrix (**K**) may be created, where κ_{ij} is the squared coherency between nodes i and j. Coherence conveys how well correlated two processes are as quantified by the cross-correlation function, which enable one to describe network evolution and co-evolution as it applies to individual nodes. Since **K** is a matrix of relations, it may be analyzed in the same manner as other networks to determine which nodes co-evolve, and which change independently. In other words, which nodes' positions in a network change similarly and which ones positions change differently. The utility of this approach was demonstrated by conducting cross-spectral analysis among 250 pairs of nation-states based on terrorism news coverage (2000–2012). The results indicate that the United States was the most central country. Also, the nations co-evolved in two large groups (the Middle East and East Asia), and numerous smaller ones. The proposed method was extended to include exogenous variables (terrorist events and oil prices) to determine the relationship between changes in these variables and changes in the terrorism news network. The proposed approach was further extended to two or more networks to examine network co-evolution. Co-evolution was demonstrated with separate data on terrorism news coverage (2011–2013) and a comparable data set on the coverage of the Arab Spring (2011–2013). The strongest ties were for a country with itself in the two networks, indicating that as it changed in the terrorist network, it changed in the Arab Spring network. Oil prices have a stronger relationship with specific nodes in the terrorist network than to nodes in the Arab Spring network.

AQ8

AQ9

Acknowledgments

This research is funded in part by the Army Research Labs (Award Number: W911NF-11-1-0405 (SPO#201118803)) and the National Science Foundation (Award Number: 1216048). An earlier draft of this manuscript was presented to the Preconference on Social and Semantic Networks in Communication Research of the International Communication Association, Seattle, WA, May, 2014.

References

Aldenderfer MS, Blashfield RK (1984) Cluster analysis. Sage, Beverly Hills

Backstrom L, Huttenlocher D, Kleinberg J, Lan X (2006) Group formation in large social networks: Membership, growth, and evolution. In: Proceedings of the 12th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining

Barnett GA, Hammond JR (2013) International media networks and terrorism shocks, paper presented to the sunbelt social networks conference, Hamburg

Barnett GA, Rice RE (1985) Network analysis in Riemann space: applications of the Galileo System to social networks. Soc Networks 7:287–322

Barnett GA, Chang HJ, Fink EL, Richards WD (1991) Seasonality in television viewing: a mathematical model of cultural processes. Communic Res 18:755–772

Barnett GA, Cho S, Choi Y (1993) Seasonality in television viewing: A dampened oscillation model. Cycles 137–141

Barnett GA, Ruiz JB, Hammond JR, Xin Z (2013) An examination of the relationship between international telecommunication networks, terrorism and global news coverage. Soc Network Anal Min 3:721–747. doi:10.1007/s13278-013-0117-9

Bastian M, Heymann S, Jacomy M (2009) Gephi: an open source software for exploring and manipulating networks. ICWSM 8:361–362

Bloomfield P (1976) Fourier analysis of time series: an introduction. Wiley, New York

Borgatti SP, Everett MG (1999) Models of core/periphery structures. Soc Networks 21:375–395

Borgatti S, Everett MG, Freeman LC (2002) UCINET 6.0 for Windows. Analytic Technologies, Harvard

Brandes U, Corman SR (2003) Visual unrolling of network evolution and the analysis of dynamic discourse? Inf Vis 2:40–50

Butts CT (2009) Revisiting the foundations of network analysis. Science 24(325):414–416

Carley KM (1999) On the evolution of social and organizational networks. In: Knoke D, Andrews SB (eds) Research in the sociology of organizations: Networks in and around organizations. JAI Press, Greenwich

Csardi G, Nepusz T (2006) The igraph software package for complex network research. InterJournal, Complex Systems, 1695(5)

Elbirt B, Barnett GA (2006) Two approaches for examining longitudinal social networks. Paper presented at the Sunbelt Social Networks Conference, Vancouver, BC

Feinerer I, Hornik K (2014) *tm: Text Mining Package.* R package version 0.5-10. http://CRAN.R-project.org/package=tm

Freeman LC (1979) Centrality in social networks: conceptual clarification. Soc Networks 1:215–239

Gliwa B, Bródka P, Zygmunt A, Saganowski S, Kazienko P, Koźlak J (2013) Different approaches to community evolution prediction in blogosphere. SNAA 2013 at ASONAM 2013, IEEE Computer Society

Gong L, Teng CY, Livne A, Brunneti C, Adamic L (2011) Coevolution of network structure and content, 3rd ACM International Conference on Web Science, Koblenz, Germany

Gottman JM (1979) Time-series analysis of continuous data in dyads. In: Lamb ME, Suomi SJ, Stephenson GR (eds) Social interaction analysis: Methodological issues. University of Wisconsin Press, Madison

Granger CWJ, Hatanaka M (1964) Spectral analysis of economic time series. Princeton University Press, Princeton

Harary F (1969) Graph Theory. Addison-Wesley, Reading

Hornik K (2014) Snowball: Snowball Stemmers. R package version 0.0-11. http://CRAN.R-project.org/package=Snowball

Hyndman RJ with Athanasopoulos G, Razbash S, Schmidt D, Zhou Z, Khan Y, Bergmeir C, Wang E (2014) forecast: Forecasting functions for time series and linear models. R package version 5.3. http://CRAN.R-project.org/package=forecast

Jenkins GM, Watt DG (1968) Spectral analysis and its application. Holden-Day, San Francisco

Jiang K, Barnett GA, Taylor LD~~,~~ (2014) Dynamics of culture frames in international news coverages: a semantic network analysis, paper presented at the Daegu (Korea) Gyeongbuk International Social Network Conference

McCulloh IA (2009) Detecting changes in a dynamic social network. Dissertation, Carnegie Mellon University, Pittsburgh

McCulloh IA, Johnson AN, Carley KM (2012) Spectral analysis of social networks to identify periodicity. J Math l Soc 36(2):80–96

Meyer D, Hornik K, Feinerer I (2008) Text mining infrastructure in R. J Stat Software 25(5):1–54

Monge PR, Contractor NS (2003) Theories of communication networks. Oxford University Press, New York

Monge P, Heiss BM, Margolin DB (2008) Communication network evolution in organizational communities. Commun Theor 18:449–477

Opsahl T (2009) Structure and evolution of weighted networks. London, University of London. Available at http://toreopsahl.com/publications/thesis/; http://toreopsahl.com/tnet/

R Core Team (2014) Foreign: read data stored by minitab, S, SAS, SPSS, Stata, Systat, Weka, dBase,… R package version 0.8-59. http://CRAN.R-project.org/package=foreign

Seary AJ, Richards WD (2003) Spectral methods for analyzing and visualizing networks: an introduction. Simon Fraser, Burnaby

Segev E, Sheafer T, Shenhav SR (2013) Is the world getting flatter? A new method for examining structural trends in the news. J Am Soc Info Sci 64:2537–2547

Singhal A, Roy A, Srivastava J (2014) Understanding co-evolution in large multi-relational social networks. IEEE IRI 2014, San Francisco

Skillicorn DB, Zheng Q, Morselli C (2014) Modelling dynamic social networks using spectral embedding. Soc Net Anal Min 4:182–203

Snijders TAB, van de Bunt GG, Steglich CE (2010) Introduction to actor based models for network dynamics. Soc Networks 32:44–60

Stoffer D (2014) Astsa: applied statistical time series analysis. R package version 1.2. http://CRAN.R-project.org/package=astsa

Toivonen R, Kovanen L, Kivelä M, Onnela J, Saramäki J, Kask K (2009) A comparative study of social network models: network evolution models and nodal attribute models. Soc Networks 31:240–254

Watson-Haigh NS, Kadarmideen HN, Reverter A (2010) PCIT: an R package for weighted gene co-expression networks based on partial correlation and information theory approaches. Bioinformatics 26(3):411–413

Weber M, Monge PR (2011) Network evolution. In: Barnett GA (ed) Encyclopedia of social networks. Sage, Los Angeles